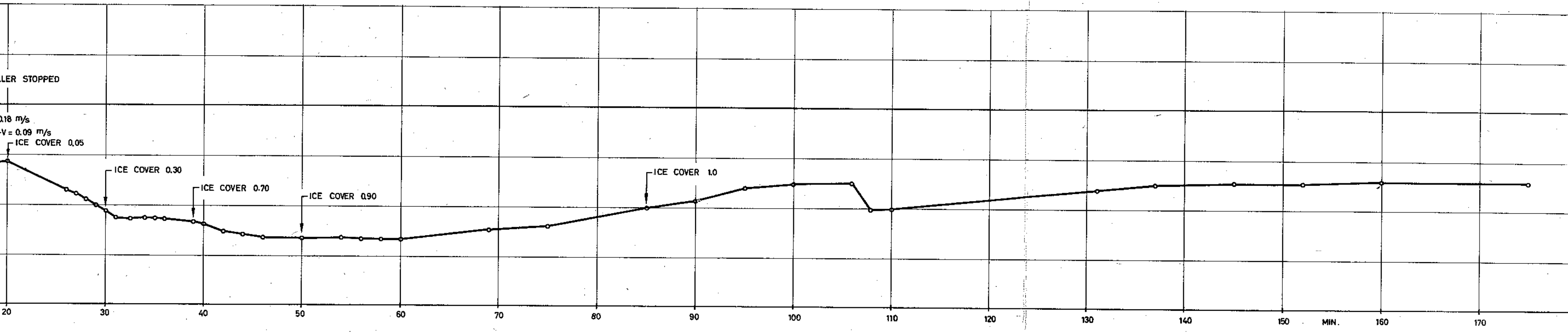

Fig. 8. Long-time history of supercooling in decaying flow.

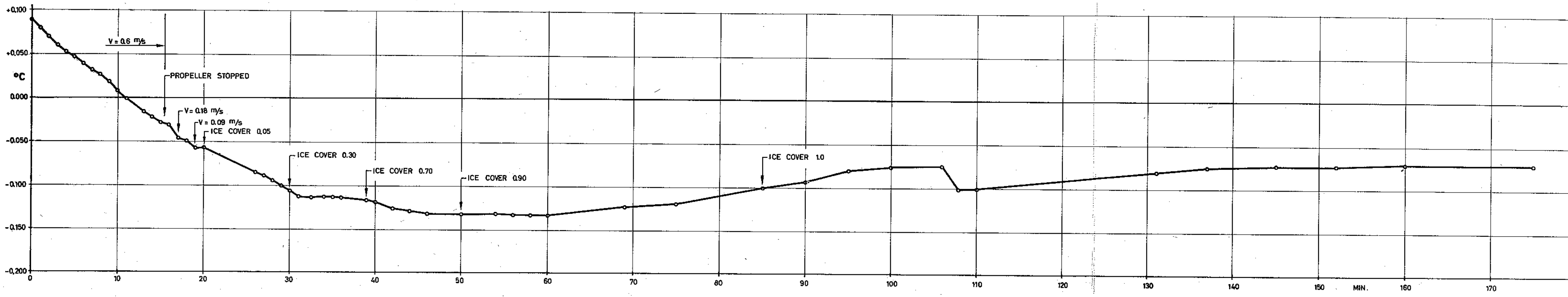

Fig. 8. Long-time history of supercooling in decaying flow.

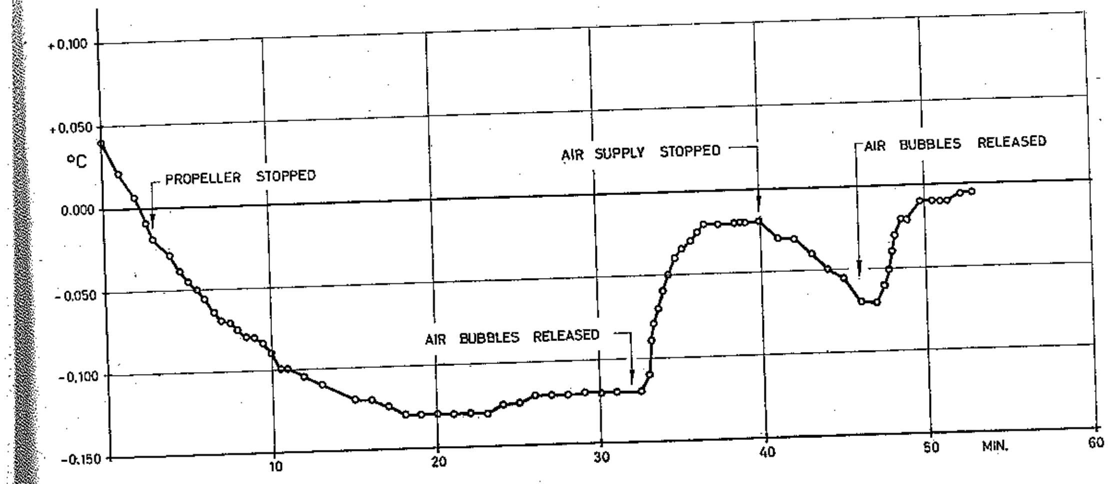

Fig. 9. Supercooling reduced by bubble-generated turbulence.

Fig. 8 shows the time history of an experiment with a decaying flow that was observed for almost three hours. The experiment is similar to that giving curve B on Fig. 6, except that the rate of heat loss is lower and that it was not interrupted. A maximum supercooling of 0.135°C was obtained after about 45 minutes. At the end of the experiment, after 165 minutes of supercooling, the water temperature was still −0.07°C.

Finally, and again with a view to engineering applications, we produced the necessary turbulence to freeze out most of the supercooling by means of air bubbles released from a perforated pipe near the bottom. Fig. 9 shows the result of a test in which a small vacuum cleaner supplied the air through a 1 m long, 10 mm brass pipe with 2 mm holes 50 mm apart. The test proceeded as that of Fig. 6B until after 30 minutes, when the temperature was −0.12°C, the air supply was switched on. The bubble-generated turbulence in a few minutes raised the water temperature to −0.02°C. After a six-minute stop of the air supply had brought the temperature down again to −0.07°C, a new air release gave a residual supercooling of about 0.005°C.

Our visual impression was that the water surface was much more agitated in the air-bubbling experiment than for a propelled flow. This fact, together with the pronounced vertical currents set up by the air bubbles, may account for the larger residual supercooling in the bubble-generated turbulence.

3. Discussion of results. The time history obtained in our experiments can be explained by due consideration of the two mechanisms involved in the freezing process: nucleation and crystal growth.

The submicroscopic phenomenon of nucleation is affected by the presence in the water of solid surfaces. The more effective surfaces are those 'wetted' easily by ice, i.e., surfaces that are strongly hydrophil with respect to ice. Information on this property can be inferred from preferential ice formation on added grains (MASON 1958) or on rods of various materials and with various coatings (MICHEL 1963).

The quality of the water, from the point of view of nucleation, is most conveniently described by the supercooling required to produce nuclei. This temperature, T_n, cannot be visually observed, however, since the nuclei are so small. The nearest observable temperature is that at which freezing starts, T_2 on Fig. 2. T_2 is probably lower than T_n by an insignificant amount.

At time t ice crystals with a total surface A have formed. On this surface, crystallization continues at a rate which is controlled by the molecular conduction of released latent heat in the immediate vicinity of the boundary:

$$L\frac{dMi}{dt}=\alpha A(T_0-T) \tag{2}$$

Here T_0 is the freezing point and T is the temperature of the water surrounding the local boundary layers. The coefficient of heat transfer α is given by

$$\alpha=\frac{k}{l}Nu \tag{3}$$

where k is the thermal conductivity of water, l is a characteristic linear dimension of the ice crystals, and Nu is the dimensionless parameter which governs the heat transfer from the crystal surfaces, the Nusselt number.

While k is a constant and l is a function of M_i, Nu depends on the geometry of the ice crystals and on the structure of the external flow. For a given geometry one can relate the Nusselt number to the Reynolds number, Re, which is a measure of the turbulence in the flow, by

$$Nu=CRe^m \tag{4}$$

Here C is a constant and $Re=Vl/v$, where V is a characteristic velocity and v is the kinematic viscosity. For laminar boundary layers dimensional considerations lead to $m=0.5$, which is experimentally confirmed. Experiments indicate that m increases with the Reynolds number, to level off at about $m=0.8$.

An example of experimentally determined, local Nusselt numbers on a circular cylinder is given in Fig. 10. It is interesting to note the high values of Nu on the downstream side of the cylinder with a turbulent wake.

We now write

$$\begin{aligned} A &\sim N^{1/3}M_i^{2/3} \\ l &\sim N^{-1/3}M_i^{1/3} \end{aligned} \tag{5}$$

where N is the number of ice crystals and M_i can be obtained by integrating (1):

$$M_i=L^{-1}[Q_s(t-t_2)-cM_w(T_2-T)] \tag{6}$$

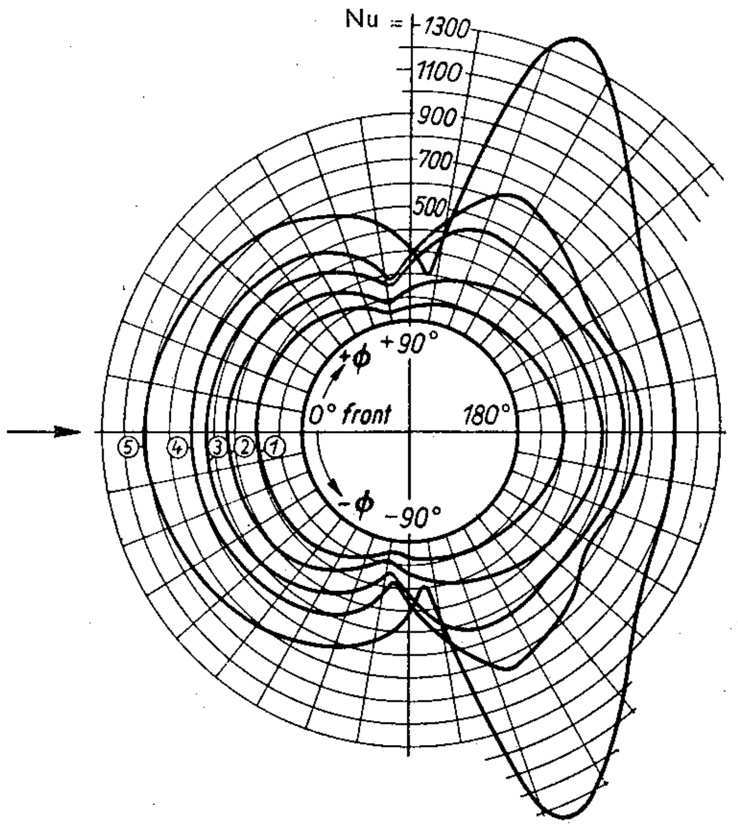

Fig. 10. Example of observed Nusselt numbers (after Schlichting).

Inserting (4) in (3), then (3) in (2) and substituting for A and l from (5) we get

$$L\frac{dM_i}{dt} \sim kM_i^{1/3}(T_0 - T)Re^m N^{2/3} \tag{7}$$

For $t = t_3$, $dT/dt = 0$ and (1) reduces to

$$L\frac{dM_i}{dt}{}_3 = Q_s \tag{8}$$

From the experimental curves on Fig. 3 we computed

$$\frac{k'}{k} = \frac{Q_s}{M_{i3}^{1/3}(T_0 - T_3)} \sim Re^m N_3^{2/3} \tag{9}$$

and obtained the figures shown in Table 1.

TABLE 1

Q_s(cal/s)	1.9	4.0	6.0	10.4	17.8
$-T_3$(10^{-2}°C)	4.6	6.8	8.2	10.2	14.1
M_{i3}(g)	2.5	3.3	4.7	4.3	4.5
k'/k	30	40	44	63	76

To the extent that Re is constant, the ratio k'/k reflects the variations in N. A slight increase in Re with increasing Q_s probably occurred, as Q_s was controlled by the air speed at the water surface. However, the added wind turbulence should be small compared to the propeller turbulence, so Re is essentially constant. The increase in k'/k with Q_s therefore must be ascribed to an increase in N.

That the number of ice crystals increased with Q_s was the immediate visual impression we got by watching the flow in the flume. Physically it is readily explained by the continued nucleation, which is primarily a function of the supercooling. As shown in Table 1, k'/k correlates well with the maximum supercooling T_3.

We shall refrain from further speculations on the actual number of ice crystals. The main point to be made here is that if we allow N to grow with T_3 or Q_s, (1) and (7) together describe time histories of water temperature similar to the observed ones:

> For constant flows, with constant Reynolds number, the $t-T$ curve has a characteristic shape, with a relatively rapid rise from a maximum supercooling, followed by an asymptotic convergence towards the freezing point (Figs. 3 and 5).
>
> For decaying flows, with decreasing Reynolds number, prolonged periods of constant supercooling are possible if the product
>
> $$Re^m N^{2/3} M_i^{1/3} = \text{const.}$$

The effect on the heat transfer of the increase in crystal size and number is then counteracted by a decrease in the turbulence of the flow. Figs. 6 and 8 show the experimental curves that suggested this line of thought.

4. Concluding remarks. We feel that the apparent stability of the unstable or metastable mixtures should be fully appreciated and not underestimated.

In our experiments we produced supercooled, meta-stable mixtures of ice and water at will. Their supercooling was a function of the rate of heat loss and of the quality of the flow.

Only a reduction of the heat loss or an increase of the heat transfer properties of the flow would reduce the supercooling. Conversely, the supercooling could be increased by an increase of the heat loss or by a decrease of the heat transfer properties of the flow.

The general relationship between the supercooling of water of constant quality and the rate of heat loss is indicated on Fig. 11, where two curves are drawn to dem-

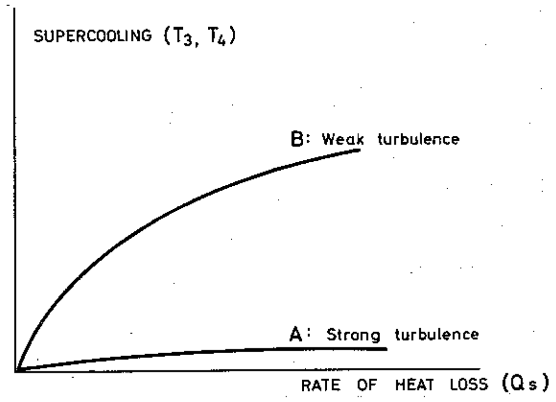

Fig. 11. Supercooling as a function of rate of heat loss.

onstrate the influence of the heat transfer properties of the flow. Similar curves, but with different numerical values, would exist for water of different quality.

The proper numerical values cannot be obtained from our experiments, because the heat loss was continually reduced at an unknown rate by the insulating effect of the ice on the water surface. Also, we have only a qualitative description of the turbulence-producing mechanisms, not a quantitative description of the flow itself. The quantification of our qualitative findings would be desirable, but we shall have to await the arrival of a suitable turbulence meter.

Field experiments and the careful interpretation of field case histories appears to be the immediate next step. We have already several case histories that seem to support our laboratory findings, but we would like to have more evidence before we give our unqualified engineering advice on river intakes.

The reason for this prudence is the rather heretic conclusion that we are forced to make on turbulence if our experimental results can be extrapolated to the field:

> In lieu of an ice cover, any cover that cuts down the heat loss from the water surface will also cut down the supercooling. The roofing of intakes proposed by Devik and Kanavin and reported by Devik (1964) is in full agreement with our experimental results and appears to be a useful innovation.
>
> One can freeze out supercooling by inducing turbulence in the flow. Therefore, if the tendency of the ice to stick is not a quality of the ice itself, but primarily the freezing of supercooled water, the stickiness of an ice-water mixture should be largely eliminated by strong turbulence. In other words, rather than design streamlined intakes with a minimum of turbulence as we do today, we should secure sufficient turbulence to exterminate the active, supercooled water and freeze it to relatively harmless slush.

The air-bubbling experiment revealed a danger of overdoing the turbulence. If the free surface becomes strongly agitated, heat losses may increase and one may end up with a larger supercooling than before. However, if the intake is covered, such a possibility is eliminated.

The lack of both quantitative information and substantial verifying field experience makes the application of these findings a delicate matter. Nevertheless, as we feel our conclusions are physically sound, we offer them here, together with the wise warning of V. E. Timonoff (1936):

> Unconsidered extrapolation of a scheme, incorrect identification of phenomena only outwardly similar, may result in a situation where experiments made in the laboratory would not be applicable to practical conditions.

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