Laboratory benchmarks for nonlinearity and separation of western boundary currents

Stefano Pierini (1), Pierpaolo Falco (1), Giovanni Zambardino (1), Thomas A. McClimans (2), Ingrid Ellingsen (2)

(1) Dipartimento di Scienze per l’Ambiente, Università di Napoli Parthenope, Naples (Italy)
(2) SINTEF, Trondheim (Norway)
• Sketch of WBC from textbook?
\[ |v| = O(V) \]
\[ |u| = O(U) \]
PV DYNAMICS FOR A CONSTANT-DEPTH, STATIONARY OCEAN FLOW

Stationarity eliminates Rossby waves and explicit dependencies on $g$ and $f_0$.

The curl of the momentum equation in the quasi-geostrophic approximation:

$$u \nabla^2 v - v \nabla^2 u + \beta v \equiv (A_H \nabla^2 - r)(v_x - u_y) ; \quad \beta = \frac{df}{dy}$$

For local WBC dynamics, surface and bottom torques are negligible ($r \sim 0$)

Dimensionless PV equation:

$$\varepsilon (u'v'_{xx} - v'u'_{xx}) + v' \equiv Ev'_{xxx}$$

$x = lx'; \quad y = Ly'; \quad u = Uu'; \quad v = Vv'; \quad \varepsilon = \frac{ul}{\beta l^2} = \left(\frac{\delta_I}{l}\right)^2; \quad E = \frac{AH}{\beta l^3} = \left(\frac{\delta_M}{l}\right)^3$

$$\delta_I = \left(\frac{ul}{\beta}\right)^{1/2} ; \quad \delta_M = \left(\frac{AH}{\beta}\right)^{1/3}$$
Gulf Stream \( \left\{ \begin{array}{l} \varepsilon \approx 0.25 \\ E \approx 5 \times 10^{-3} \end{array} \right. \\
\) Rotating Tank Exp. B \( (A = 10^{-4} \text{ m}^2/\text{s}) \) \( \left\{ \begin{array}{l} \varepsilon \approx 0.28 \\ E \approx 5.7 \times 10^{-3} \\ (\delta_* \approx 5 \text{ cm}) \end{array} \right. \\
\)

Rotating Tank \( (\text{water}) \) Exp. B \( \left\{ \begin{array}{l} \varepsilon \approx 0.28 \\ E \approx 5.7 \times 10^{-5} \\ (\delta_*' \approx 0.5 \text{ cm}) \end{array} \right. \\
\)

dynamically similar

comparable flow at distances from the boundary larger than \( \delta_* \)

\[ \delta_* = \left( \frac{A \delta_I}{U} \right)^{1/2} \approx \frac{\delta_I}{\sqrt{\text{Re}}} \ll \delta_I. \]
CLASSICAL SOLUTIONS

(Field data) (Linear) (Nonlinear)


Morgan (1956)
Fig. 5 in PEA'08 with comments on the different BL's – criticized for being too strong.

Exp. A

Exp. C

Exp. B

Exp. D
Fig. 4 from PEA'10

(a) $T=30\ s$

$u_F =$

- 0.5 mm s$^{-1}$ (1)
- 1 mm s$^{-1}$ (2)
- 2 mm s$^{-1}$ (3)
- 4 mm s$^{-1}$ (4)
- 8 mm s$^{-1}$ (5)
- 20 mm s$^{-1}$ (6)
- 30 mm s$^{-1}$ (7)

(b) $T=60\ s$

$u_F =$

- 0.5 mm s$^{-1}$ (1)
- 1 mm s$^{-1}$ (2)
- 2 mm s$^{-1}$ (3)
- 4 mm s$^{-1}$ (4)
- 8 mm s$^{-1}$ (5)
- 20 mm s$^{-1}$ (6)
•Fig. 5 from PEA'10 with comments on the different theories

- Dynamically similar Gulf Stream flow (away from the viscous boundary layer)

\[ x_s \propto u_p^\alpha \]

- Charney \((\alpha=1/2)\)

- Munk \((\alpha=0)\)

- \( Re = 3040 \)

- \( Re = 380 \)

- \( Re = 160 \)

- \( Re = 70 \)

- \( Re = 1 \)

- \( Re = 0.7 \)

- \( Re = 0.1 \)

\( x_s \) (cm) vs. \( u_p \) (cm/s)
• Sketch of WBC from textbook?
Fig. 13 from PEA'10 with discussion on different theories.
SUMMARY

New results for the "nonlinear Munk range".
Modest changes due to shelf slope.
Plausible explanation for separation at Cape Hatteras?

Thank you for your attention and thanks to the EC’s 6th Framework Programme "Integrated Infrastructure Initiative HYDRALAB III”, Contract No. 022441